



## Corrigendum to “Weighted $L^p$ estimates for the elliptic Schrödinger operator” [*Electron. J. Qual. Theory Differ. Equ.* 2014, No. 33, 1–13]

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**Abstract.** An error in Lemma 1.5 in “Weighted  $L^p$  estimates for the elliptic Schrödinger operator” [*Electron. J. Qual. Theory Differ. Equ.* 2014, No. 33, 1–13] is corrected.

**Keywords:** weighted, regularity,  $L^p$  estimates, elliptic, Schrödinger operator.

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The author regrets to inform that Lemma 1.5 in [1] is not fully correct. Since the main results in [1] eventually rely on this lemma, we shall give the following new lemma instead of the wrong Lemma 1.5 in [1] and then put it behind Lemma 1.9 in [1].

**Lemma 1.9.** Assume that  $w \in A_q$  for some  $q > 1$ .

(1) There exists a positive constant  $q_1 \in (1, q)$  such that

$$w \in A_{q_1}.$$


(2) Let  $q_2 = \frac{q}{q_1} \in (1, q)$ . Then we have

$$L_w^q(B_r) \subset L^{q_2}(B_r).$$

*Proof.* Since  $w \in A_q$ , from Definition 1.3 in [1] we have

$$\begin{aligned} & \left( \int_{B_r} w(x) dx \right) \left( \int_{B_r} w(x)^{\frac{-1}{q-1}} dx \right)^{q-1} \\ &= \left( \int_{B_r} \left( w(x)^{\frac{-1}{q-1}} \right)^{1-q} dx \right) \left( \int_{B_r} w(x)^{\frac{-1}{q-1}} dx \right)^{q-1} \\ &= \left[ \left( \int_{B_r} \left( w(x)^{\frac{-1}{q-1}} \right)^{-\frac{1}{\frac{q}{q-1}-1}} dx \right)^{\frac{q}{q-1}-1} \left( \int_{B_r} w(x)^{\frac{-1}{q-1}} dx \right) \right]^{q-1} \leq C \end{aligned} \quad (1.1)$$

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for any balls  $B_r$  in  $\mathbb{R}^n$ , which implies that  $w(x)^{\frac{-1}{q-1}} \in A_{\frac{q}{q-1}}$ . Therefore, from Lemma 1.8 in [1] we have

$$\left( \int_{B_r} w(x)^{-\frac{1+\epsilon'_0}{q-1}} dx \right)^{\frac{1}{1+\epsilon'_0}} \leq C \int_{B_r} w(x)^{\frac{-1}{q-1}} dx \quad (1.2)$$

for some  $\epsilon'_0 \in (0, 1)$ . Let

$$q_1 = 1 + \frac{q-1}{1+\epsilon'_0} \in (1, q).$$

Then from (1.2) and the fact that  $w \in A_q$  we find that

$$\begin{aligned} & \left( \int_{B_r} w(x) dx \right) \left( \int_{B_r} w(x)^{\frac{-1}{q_1-1}} dx \right)^{q_1-1} \\ &= \left( \int_{B_r} w(x) dx \right) \left( \int_{B_r} w(x)^{-\frac{1+\epsilon'_0}{q-1}} dx \right)^{\frac{q-1}{1+\epsilon'_0}} \\ &\leq C \left( \int_{B_r} w(x) dx \right) \left( \int_{B_r} w(x)^{-\frac{1}{q-1}} dx \right)^{q-1} \leq C, \end{aligned} \quad (1.3)$$

which implies that  $w \in A_{q_1}$ . Furthermore, if  $f \in L^q_w(B_r)$ , then from Hölder's inequality and (1.3) we have

$$\begin{aligned} \int_{B_r} |f|^{\frac{q}{q_1}} dx &= \int_{B_r} |f|^{\frac{q}{q_1}} w(x)^{\frac{1}{q_1}} w(x)^{-\frac{1}{q_1}} dx \\ &\leq \left( \int_{B_r} |f|^q w(x) dx \right)^{\frac{1}{q_1}} \left( \int_{B_r} w(x)^{-\frac{1}{q_1-1}} dx \right)^{1-\frac{1}{q_1}} \\ &\leq C \left( \int_{B_r} |f|^q w(x) dx \right)^{\frac{1}{q_1}} \left( \frac{|B_r|}{w(B_r)} \right)^{\frac{1}{q_1}} \leq C, \end{aligned}$$

since  $w \in L^1_{\text{loc}}(\mathbb{R}^n)$  and  $w > 0$  almost everywhere. This finishes our proof by choosing  $q_2 = \frac{q}{q_1} \in (1, q)$ .  $\square$

We shall add the following sentences in front of “Next, we shall prove the following important result” in [1], page 5, line -6:

“Assume that  $w \in A_p$ . Then from Lemma 1.9 (1) (see above) we find that

$$w \in A_{p_1} \quad \text{for some } p_1 \in (1, p).” \quad (1.4)$$

Moreover, we shall change “Assume that  $1 < q < p$ ” in Lemma 2.2 of [1] to “Assume that  $w \in A_p$ ” and add the sentence “where  $q = \frac{p}{p_1} \in (1, p)$  and  $p_1$  is defined in (1.4),” behind (2.2) in [1]. Furthermore, we shall change “Assume that  $1 < q < p$  and  $w \in A_p$ ” in Corollary 2.3 of [1] to “Assume that  $w \in A_p$ ” and add the sentence “where  $q = \frac{p}{p_1} \in (1, p)$  and  $p_1$  is defined in (1.4),” behind (2.8) in [1].

The author would like to apologize for any inconvenience caused.

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## References

- [1] F. YAO, Weighted  $L^p$  estimates for the elliptic Schrödinger operator, *Electron. J. Qual. Theory Differ. Equ.* **2014**, No. 33, 1–13. [MR3250024](#); [url](#)